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Radiative neutron β–decay in effective field theory

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Abstract

We consider radiative β–decay of the neutron in heavy baryon chiral perturbation theory, with an extension including explicit Δ degrees of freedom. We compute the photon energy spectrum as well as the photon polarization; both observables are dominated by the electron bremsstrahlung contribution. Nucleon-structure effects not encoded in the weak coupling constants g_A and g_V are determined at next-to-leading order in the chiral expansion, and enter at the $\mathcal{O}(0.5\%)$-level, making a sensitive test of the Dirac structure of the weak currents possible.

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1. Experimental studies of β–decay at low energies have played a crucial role in the rise of the Standard Model (SM) [1]. In recent years, continuing, precision studies of neutron β–decay have been performed, to better both the determination of the neutron lifetime and of the correlation coefficients. Taken in concert, these measurements yield the weak coupling constants g_V and g_A [#5]; g_V, in turn, yields the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{ud} and, with the empirical values of V_{us} and V_{ub}, the most precise test of the unitarity of the CKM matrix. As the neutron measurements improve, further SM tests become possible, such as a precision test of the CVC hypothesis, as well as of the absence of second-class currents, yielding, generally, improved constraints on the appearance of non-$V-A$ currents [2].

To realize a SM test to a precision of $\sim 1\%$ or better requires the application of radiative corrections [3]. For example, a new measurement of the A correlation coefficient in neutron β–decay, with the world average values for the neutron lifetime, V_{us}, and V_{ub} [4], yields $1-\sum_{j=d,s,b}|V_{uj}|^2 = 0.0083(28)$ [5], indicating a deviation of 3σ from CKM unitarity. The significance of the deviation from unitarity depends on the radiative corrections and their surety. One component of such, the "outer" radiative correction, is captured by electromagnetic interactions with the charged, final-state particles, in the limit in which their structure is neglected. In this, neutron radiative β–decay enters, and we consider it explicitly. We find neutron radiative β–decay interesting in its own right, though the process has yet to be observed — only an upper bound exists [6]. Anticipating its measurement, however, and as the precision of such improves, we can (i) hope to effect an alternative determination of the weak couplings g_V and g_A. The photon energy spectrum in neutron radiative β–decay in leading order is characterized by contributions proportional to $g_V^2 + 3g_A^2$ and to $g_V^2 - g_A^2$, so that g_V and g_A can be determined, though $(g_V^2 - g_A^2)/(g_V^2 + 3g_A^2) \sim 0.10$. (ii) We can study the hadron matrix elements in subleading order, $\mathcal{O}(1/M)$, with M the neutron mass. Here, we note the connection to radiative muon capture on the proton, which permits the determination of the induced pseudoscalar coupling constant g_P. The only measurement thus far of radiative muon capture [7] yields a result for g_P which is significantly at odds with the chiral perturbation theory prediction [8]. For recent reviews containing extensive discussions of possible resolutions to this problem, see Refs. [9, 10]. The same hadronic matrix elements, calculated in Ref. [11] in the framework of an effective field theory (EFT) of nucleons, pions, and external sources (and Δs), appear in radiative neutron capture, albeit at much smaller momentum transfers. Consequently, one could integrate out the Δs and even the pions from the EFT, resulting in an equally precise calculational tool but with no direct access to and thus test of the chiral structure of QCD at low energies.[#6] (iii) We can use neutron radiative β–decay to test the Dirac structure of the weak current, through the determination of the circular polarization of the associated photon [12]. As recognized shortly after the discovery of parity violation in β–decay [13], the photon emitted in associated radiative processes should be circularly polarized [14, 15]. In integrating over the phase space, it becomes apparent that the photon becomes $\sim$100% polarized only when its energy grows large; in our explicit calculations we confirm that the predictions of Ref. [15] for internal bremsstrahlung, i.e., for radiative orbital electron capture of S-state electrons, are germane to radiative β–decay as well. This prediction follows from a perfectly right-handed anti-neutrino and from the absence of scalar, tensor, and pseudoscalar interactions in leading order.

In this letter, we perform a systematic analysis of neutron radiative β–decay in the framework of heavy baryon chiral perturbation theory (HBCHPT) [16, 17, 18] and in the small scale expansion (SSE) [19], including all terms in $O(1/M)$, i.e., at next–to–leading order (NLO) in the small parameter ϵ. Here, ϵ collects all the small external momenta and quark (pion) masses, relative to the heavy baryon

[#5] Precise definitions of the various form factors and couplings follow.

[#6] Alternatively, one could use a nonrelativistic EFT for the calculation and then perform matching to the amplitudes evaluated in heavy baryon chiral perturbation or in the small scale expansion. We prefer, however, to work with an EFT including explicit pions (and Δs).

mass M, which appear when HBCHPT is utilized; in case of the SSE, such is supplemented by the $\Delta(1232)$–nucleon mass splitting, relative to the nucleon mass, as well. These systematic EFTs allows one to calculate the recoil-order corrections in a controlled way. In order to assess the size of the recoil-order corrections, we compare with the pioneering work of Ref. [12], in which such effects have been neglected. In that calculation, the standard parameterization of the hadronic weak current in terms of the weak coupling constants suffices to capture the hadron physics. No reference to photon emission from the effective four–fermion vertex is found in these papers. Here, we include all terms in $O(1/M)$, utilizing the framework of HBCHPT and the SSE for the actual calculations. In fact, the pertinent two– and four–point functions can be taken directly from Ref. [11], after relabelling the momenta and such.

2. First, we collect some definitions for the process under consideration,

$$n(p) \to p(p') + e^-(l_e) + \bar{\nu}_e(l_\nu) + \gamma(k) \;, \tag{1}$$

where p, p', l_e, l_ν, and k denote the four–momentum of the neutron, proton, electron, anti-neutrino, and photon, respectively — we denote the photon energy by ω. In the static approximation for the W^--boson, which is appropriate here, the matrix element for radiative neutron β–decay decomposes into two pieces,

$$\begin{aligned}\mathcal{M}(n \to p e^- \bar{\nu}_e \gamma) &= \langle \bar{\nu}_e \, e^- | J^-_\mu | 0 \rangle \, i \, \frac{g^{\mu\nu}}{M_W^2} \langle p | \mathcal{T}\,(V \cdot \epsilon^* V^+_\nu - V \cdot \epsilon^* A^+_\nu) | n \rangle \\ &\quad + \langle \bar{\nu}_e \, e^- \, \gamma | J^-_\mu | 0 \rangle \, i \, \frac{g^{\mu\nu}}{M_W^2} \langle p | V^+_\nu - A^+_\nu | n \rangle \;,\end{aligned} \tag{2}$$

in terms of the leptonic weak current (J^-), as well as the hadronic vector (V) and axial–vector (A) currents; ϵ_μ is the photon polarization vector. For later use, we introduce the Fermi constant G_F via $G_F = g_2^2\sqrt{2}/(8M_W^2)$, where M_W is the W–boson mass and g_2 is the usual $\mathrm{SU}(2)_L$ gauge coupling constant. The first term in Eq. (2) includes bremsstrahlung from the proton, as well as radiation from the effective weak vertex, which includes radiation from the pion in flight, whereas the second term corresponds to bremsstrahlung from the electron in the final state. These contributions are illustrated in Fig. 1.

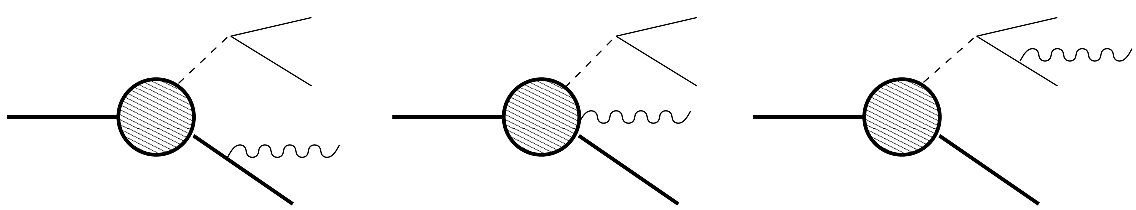

Figure 1: Contributions to $n \to p e^- \bar{\nu}_e \gamma$ in $\mathcal{O}(G_F)$; heavy lines denote nucleons, light lines denote leptons, wiggly lines denote photons, and the shaded circle denotes the effective weak vertex.

We now discuss the leptonic and hadronic matrix elements appearing in Eq. (2). The pertinent leptonic current matrix elements are

$$\langle \bar{\nu}_e \, e^- | J^-_\mu \, | 0 \rangle = -i \, \frac{g_2}{\sqrt{8}} \, \bar{u}_e(l_e) \gamma_\mu (1 - \gamma_5) v_\nu(l_\nu) \,, \tag{3}$$

$$\langle \bar{\nu}_e \, e^- \, \gamma | J^-_\mu \, | 0 \rangle = i \, \frac{g_2 e}{\sqrt{8}} \, \bar{u}_e(l_e) \left(\frac{2\epsilon^* \cdot l_e - \not{k}\, \not{\epsilon}^*}{2 l_e \cdot k} \right) \gamma_\mu (1 - \gamma_5) v_\nu(l_\nu) \;, \tag{4}$$

whereas the most general form of the hadronic weak current matrix elements, consistent with the $V-A$ structure of the SM, is [20]

$$\langle p\,|V_\mu^+\,|n\rangle \;=\; -i\,\frac{g_2}{\sqrt{8}}\,\bar{u}_p(p')\left[F_1(q^2)\gamma_\mu - \frac{i}{2M}F_2(q^2)\sigma_{\mu\nu}q^\nu + \frac{F_3(q^2)}{2M}q_\mu\right]u_n(p)\;, \tag{5}$$

$$\langle p\,|A_\mu^+\,|n\rangle \;=\; -i\,\frac{g_2}{\sqrt{8}}\,\bar{u}_p(p')\left[G_1(q^2)\gamma_\mu\gamma_5 - \frac{i}{2M}G_2(q^2)\sigma_{\mu\nu}\gamma_5 q^\nu - \frac{G_3(q^2)}{2M}q_\mu\gamma_5\right]u_n(p)\;, \tag{6}$$

with $\sigma^{\mu\nu} = i[\gamma^\mu,\gamma^\nu]/2$ and $q_\mu \equiv (p-p')_\mu$. We note that Eqs.(3-6) employ conventional spinors, satisfying, e.g., $\sum_s u_e(l,s)\bar{u}_e(l,s) = \not{l} + m_e$. The weak coupling constants g_V and g_A, which appear in leading order, are defined via $F_1(0) \equiv g_V$ and $G_1(0) \equiv g_A$. We note that $g_A/g_V \equiv \lambda = 1.2670 \pm 0.0030$ as determined from the A correlation coefficient in neutron β–decay [21]. In the SM, under an assumption of isospin symmetry, the CVC hypothesis relates the weak vector form factors to the (electromagnetic) Dirac and Pauli form factors; we recall that the Dirac form factor is unity at $q^2 = 0$, so that $g_V \equiv (1+\Delta_R^V)^{1/2}V_{ud}$, where Δ_R^V is a small, radiative correction [3] and V_{ud} is a Cabibbo-Kobayashi-Maskawa (CKM) matrix element. Moreover, the CVC hypothesis and isospin symmetry determines the weak magnetism term, namely, that $F_2(0)/F_1(0) = \kappa_v$, where $\kappa_v = 3.706$ is the isovector nucleon anomalous magnetic moment; we have neglected the possibility of an additional radiative correction which is not common to F_1 and F_2. The second-class current contributions $F_3(q^2)$ and $G_2(q^2)$ vanish at $q^2 = 0$ in this limit, so that henceforth we omit any discussion of them entirely. Isospin is an approximate symmetry of the SM, so that corrections to these expectations, save that of $F_1(0)$, are of $\mathcal{O}(R)$, where $R \approx (M-M')/M_N$, noting that M and M' are the neutron and proton mass, respectively, with $M_N \equiv (M+M')/2$ the average neutron-proton mass. Such corrections, however, are systematically of higher order in our power counting scheme and thus can be neglected to the order, $\mathcal{O}(1/M^2)$, in which we work. Usually the non–relativistic reduction of Eqs.(5,6) is done in the Breit frame. Here we give the non–relativistic strong matrix elements in the rest frame of the neutron where our calculation is done:

$$\begin{aligned}
\langle p|\,V_\mu^+\,|n\rangle \;=\; & -i\,\frac{g_2}{\sqrt{8}}\,\mathcal{N}'\,\bar{p}_v(p')\left\{\left(\frac{2M}{E'+M}F_1(q^2) - \frac{E'-M}{E'+M}F_2(q^2)\right)v_\mu\right.\\
& -\left[\frac{1}{E'+M}\left(F_1(q^2)+F_2(q^2)\right) - \frac{1}{2M}F_2(q^2)\right]q_\mu\\
& \left.-\frac{2}{E'+M}\,[S_\mu, S\cdot q]\left(F_1(q^2)+F_2(q^2)\right)\right\}n_v(0)\;,
\end{aligned} \tag{7}$$

$$\langle p|\,A_\mu^+\,|n\rangle \;=\; -i\,\frac{g_2}{\sqrt{8}}\,\mathcal{N}'\,\bar{p}_v(p')\left\{G_1(q^2)\left[2\,S_\mu + \frac{2\,S\cdot q\,v_\mu}{E'+M}\right] + G_3(q^2)\,\frac{S\cdot q\,q_\mu}{M\,(E'+M)}\right\}n_v(0), \tag{8}$$

where we expand Eqs. (7,8) to $\mathcal{O}(1/M^2)$ in all applications. Note that $\mathcal{N}'$ is the usual normalization factor of the proton wave function, $\mathcal{N}' = \sqrt{(E'+M)/2M}$ and E' is the proton energy. We have employed non-relativistic nucleon spinors, with normalization $\sum_\sigma n_v(r,\sigma)\bar{n}_v(r,\sigma) = P_v^+(1+v\cdot r/(2M))$, where $P_v^+ \equiv (1+\not{v})/2$. We make use of the fact that in HBCHPT, and in the SSE, the nucleon four–momentum p_μ is written as $p_\mu = Mv_\mu + r_\mu$, with v_μ the fixed four–velocity, subject to the constraint $v^2 = 1$ and $r\cdot v \ll M$. Furthermore, S_μ is the nucleon's (Pauli-Lubanski) spin vector with $v\cdot S = 0$. The explicit form of the four form factors appearing in the above equations, expanded to next-to-leading order in HBCHPT and in the SSE, can be taken from Refs.[17, 22, 23] for HBCHPT and from Ref.[22] in the SSE. At the small momentum transfers of current interest, however, it suffices to replace the form factors with their values at zero q^2, though we do employ $G_3(q^2)/M = 4g_{\pi NN}F_\pi/(m_\pi^2-q^2) - 2\lambda M r_A^2/3$, where the radiative corrections implicit to the use of λ in this case are without numerical consequence. For reference, the induced pseudoscalar coupling constant, g_P, is $g_P \equiv G_3(-0.88m_\mu^2)/2M$ with m_μ

the muon mass. We now turn to the vector–vector (VV) and vector–axial (VA) correlators, which we need to $\mathcal{O}(p^2)$ in HBCHPT, or to $\mathcal{O}(\epsilon^2)$ in the SSE. Working in the Coulomb gauge $\epsilon^* \cdot v = 0$ for the photon and making use of the transversality condition $\epsilon^* \cdot k = 0$, we find

$$
\begin{aligned}
\langle p|\, \mathcal{T}\, V\cdot\epsilon^* V^+_\mu \,|n\rangle^{(2)} &= -i\,\frac{g_2 g_V\, e}{\sqrt{8}}\,\bar{p}_v(r')\left\{-\frac{(1+\kappa_v)}{M}\,[S_\mu, S\cdot\epsilon^*] - \frac{1}{2M}\,\epsilon^*_\mu \right.\\
&\left. \qquad -\frac{1}{M\omega}\,v_\mu\left[(1+\kappa_v)\,[S\cdot\epsilon^*, S\cdot k] - \epsilon^*\cdot r'\right] + \mathcal{O}\left(\frac{1}{M^2}\right)\right\} n_v(r)\ , \qquad (9)\\
\langle p|\, \mathcal{T}\, V\cdot\epsilon^* A^+_\mu \,|n\rangle^{(2)} &= -i\,\frac{g_2 g_V\, e}{\sqrt{8}}\,\bar{p}_v(r')\left\{-2\lambda\,\frac{S\cdot(r'-r)}{(r'-r)^2 - m_\pi^2}\left[\frac{2\,\epsilon^*\cdot(l_e+l_\nu)\,(l_e+l_\nu)_\mu}{(l_e+l_\nu)^2 - m_\pi^2} - \epsilon^*_\mu\right]\right.\\
&\quad +\frac{\lambda}{M}\,\frac{(v\cdot r' - v\cdot r)\,S\cdot(r+r')}{(r'-r)^2 - m_\pi^2}\left[\frac{2\,\epsilon^*\cdot(l_e+l_\nu)\,(l_e+l_\nu)_\mu}{(l_e+l_\nu)^2 - m_\pi^2} - \epsilon^*_\mu\right]\\
&\quad -2\,\lambda\left[1 + \left(\frac{v\cdot l_e + v\cdot l_\nu}{2M}\right)\right]\frac{S\cdot\epsilon^*\,(l_e+l_\nu)_\mu}{(l_e+l_\nu)^2 - m_\pi^2} + \frac{\lambda}{M}\,S\cdot\epsilon^*\,v_\mu\\
&\quad -\frac{\lambda}{M}\left[\frac{(2+\kappa_s+\kappa_v)\,[S\cdot\epsilon^*, S\cdot k]\,S^\alpha}{\omega} + \frac{(\kappa_v-\kappa_s)\,S^\alpha\,[S\cdot\epsilon^*, S\cdot k]}{\omega}\right.\\
&\left.\left.\qquad -\frac{2\,S^\alpha\epsilon^*\cdot r'}{\omega}\right]\left[g_{\mu\alpha} - \frac{(l_e+l_\nu)_\alpha(l_e+l_\nu)_\mu}{(l_e+l_\nu)^2 - m_\pi^2}\right] + \mathcal{O}\left(\frac{1}{M^2}\right)\right\} n_v(r)\ , \qquad (10)
\end{aligned}
$$

with $\omega = v\cdot k$. Also, m_π is the charged pion mass, and $e = |e|$ is the elementary charge. Turning to the SSE, we note that the vector–vector correlator is free of Δ effects to $\mathcal{O}(\epsilon^2)$, so that the leading $\Delta(1232)$ effect appears only in the vector–axial correlator, given by

$$
\begin{aligned}
\langle p|\mathcal{T}\, V\cdot\epsilon^* A^+_\mu|n\rangle^{(2),\Delta} &= -i\,\frac{g_2 g_V\, e}{\sqrt{8}}\,\bar{p}_v(r')\left\{-\frac{g_{\pi N\Delta} b_1}{3\,M}\right.\\
&\quad\times\left[\frac{2\Delta\,[k^\alpha S\cdot\epsilon^* - \omega\, v^\alpha S\cdot\epsilon^* - \epsilon^{*\,\alpha} S\cdot k]}{\Delta^2 - \omega^2} + \frac{4\,[S\cdot\epsilon^*, S\cdot k]\,S^\alpha}{3\,(\Delta-\omega)}\right.\\
&\left.\left.\quad -\frac{4\,S^\alpha\,[S\cdot\epsilon^*, S\cdot k]}{3\,(\Delta+\omega)}\right]\left[g_{\mu\alpha} - \frac{(l_e+l_\nu)_\alpha(l_e+l_\nu)_\mu}{(l_e+l_\nu)^2 - m_\pi^2}\right] + \mathcal{O}\left(\frac{1}{M^2}\right)\right\} n_v(r)\ , \qquad (11)
\end{aligned}
$$

where $g_{\pi N\Delta} = 1.05$ and $b_1 = 12.0$ are the leading strong and electromagnetic coupling constants in the coupled $N\Delta\pi\gamma$ system [11], noting that $\Delta \equiv M_\Delta - M$. We neglect the radiative correction to this contribution, as the contribution itself is extremely small. One can easily check from the continuity equations satisfied by the correlators that gauge invariance is satisfied in the above equations [11]. Note that the (2) superscript explicitly indicates that we report the matrix elements in NLO.

3. Let us compare our matrix elements with those of Ref. [12]. As Eqs. (9-11) make apparent, only the electron bremsstrahlung contribution makes an $\mathcal{O}(1)$ contribution to neutron radiative β–decay. Such a result is at odds with Ref. [12] and, indeed, with the literature on "outer" radiative corrections [3] in neutron β–decay. In these papers there is an $\mathcal{O}(1)$ contribution from proton bremsstrahlung as well. The source of the *apparent* discrepancy can readily be found. The form of the decay amplitude for neutron radiative β–decay, as follows from computing the bremstrahlung contributions in QED [12], is

$$
\begin{aligned}
\mathcal{M} \;=\; \frac{e g_V G_F i}{\sqrt{2}}&\left\{\bar{u}_e(l_e)\frac{(2l_e\cdot\epsilon^* + \not{\epsilon}^*\not{k})}{2l_e\cdot k}\gamma_\rho(1-\gamma_5)v_\nu(l_\nu)\bar{u}_p(p')\gamma^\rho(1-\lambda\gamma_5)u_n(p)\right.\\
&\left. -\bar{u}_e(l_e)\gamma_\rho(1-\gamma_5)v_\nu(l_\nu)\bar{u}_p(p')\frac{(2p'\cdot\epsilon^* + \not{\epsilon}^*\not{k})}{2p'\cdot k}\gamma^\rho(1-\lambda\gamma_5)u_n(p)\right\}\ . \qquad (12)
\end{aligned}
$$

The QED treatment neglects photon emission from the effective weak vertex; it is correct in leading order in $1/M$ only. Consequently, we consider $|\mathcal{M}|^2$ here in leading order only. Note that for each photon polarization state $p' \cdot \epsilon^*/p' \cdot k$ is of $\mathcal{O}(1/M)$, so that the proton bremsstrahlung contribution is also of $\mathcal{O}(1/M)$ — and thus negligible. However, in effecting the photon polarization sum, the gauge invariance of QED also permits the replacement $\sum_\sigma \epsilon^*_\mu(\sigma)\epsilon_\nu(\sigma) \to -g_{\mu\nu}$. This suggests that the $p' \cdot \epsilon'/p' \cdot k$ term, when squared and summed over the photon helicity, yields a contribution of $\mathcal{O}(1)$. This is, indeed, what happens upon explicit calculation. Employing lepton and hadron tensors, the square of the matrix element can be written as

$$\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{e^2 g_V^2 G_F^2}{2} \left(\frac{1}{(l_e \cdot k)^2} L^{\text{ee}}_{\rho\delta} H^{\rho\delta} + \frac{1}{M'^2 \omega^2} L^{\rho\delta} H^{\text{ee}}_{\rho\delta} - \frac{1}{M'\omega(l_e \cdot k)} M^{\text{ee, mixed}} \right) , \tag{13}$$

where we have retained only the leading expression in each term. Employing the $g_{\mu\nu}$ replacement for the photon helicity sum, we find

$$\begin{aligned}
L^{\text{ee}}_{\rho\delta} H^{\rho\delta} &= -64 M M' (m_e^2 - l_e \cdot k) \left((1+3\lambda^2) E_\nu (E_e + \omega) + (1-\lambda^2)(\mathbf{l_e} \cdot \mathbf{l}_\nu + \mathbf{l}_\nu \cdot \mathbf{k}) \right) , & (14)\\
L^{\rho\delta} H^{\text{ee}}_{\rho\delta} &= -64 M (M')^3 \left((1+3\lambda^2) E_\nu E_e + (1-\lambda^2) \mathbf{l}_\nu \cdot \mathbf{l_e} \right) , & (15)\\
M^{\text{ee, mixed}} &= -64 M (M')^2 \left((1+3\lambda^2) E_\nu (2E_e^2 + E_e \omega - k \cdot l_e) \right. \\
&\quad \left. + (1-\lambda^2)(2E_e \mathbf{l}_\nu \cdot \mathbf{l_e} + E_e \mathbf{l}_\nu \cdot \mathbf{k}) \right) , & (16)
\end{aligned}$$

identical to the result of Ref. [12], save for an overall sign. We have checked that this result is identical to that obtained using the leading contribution from the $L^{\text{ee}}_{\rho\delta} H^{\rho\delta}$ term exclusively, after explicitly summing over the photon polarization states. Equation (12) and Eqs. (2-6) are consistent to leading order in $1/M$. Furthermore, the leading contribution to the outer radiative corrections in neutron β–decay is also from electron bremsstrahlung, as calculated here, complemented by the photon exchange graph — for a recent attempt at calculating radiative corrections to neutron β–decay within EFT, see Ref. [24].

Noting the normalization of the nonrelativistic spinors [11], the total decay rate is given by

$$\Gamma = \frac{1}{(2\pi)^8} \int d^3\mathbf{p}'\, d^3\mathbf{l_e}\, d^3\mathbf{l}_\nu\, d^3\mathbf{k}\, \frac{M'}{E'} \frac{1}{2E_\nu} \frac{1}{2E_e} \frac{1}{2E_\gamma} \sum_{\text{spins}} |\mathcal{M}|^2\, \delta^{(4)}(p - p' - l_e - l_\nu - k) , \tag{17}$$

or

$$\Gamma = \frac{M'}{8(2\pi)^8} \int |\mathbf{l_e}| \omega d\omega\, dE_e\, d\Omega_e d\Omega_k\, d\Omega_\nu \left[\frac{\Theta(M - E_e - E_\nu - \omega)\, E_\nu \sum_{\text{spins}} |\mathcal{M}|^2 |_{p'=p-l_e-l_\nu-k}}{|M - E_e - \omega + \mathbf{l_e} \cdot \mathbf{n}_\nu + \mathbf{k} \cdot \mathbf{n}_\nu|} \right] , \tag{18}$$

where $\mathbf{n}_\nu \equiv \hat{\mathbf{l}}_\nu$ and

$$E_\nu = \frac{M^2 + m_e^2 - M'^2 - 2M(E_e + \omega) + 2E_e\omega - 2\mathbf{l_e} \cdot \mathbf{k}}{2(M - E_e - \omega + \mathbf{l_e} \cdot \mathbf{n}_\nu + \mathbf{k} \cdot \mathbf{n}_\nu)} . \tag{19}$$

To complete the integration over the four-particle phase space, we let $\hat{\mathbf{l}}_\mathbf{e}$ define the $\mathbf{z}$–direction, so that $\hat{\mathbf{k}} \cdot \hat{\mathbf{l}}_\mathbf{e} \equiv x_k$ and $\mathbf{n}_\nu \cdot \hat{\mathbf{l}}_\mathbf{e} \equiv x_\nu$. Thus Eq. (18) can be cast in the form

$$\begin{aligned}
\Gamma(\omega^{\min}) &= \frac{M'}{4(2\pi)^6} \int_{\omega^{\min}}^{\omega^{\max}} \omega d\omega \int_{m_e}^{E_e^{\max}(\omega)} |\mathbf{l_e}| dE_e \int_{x_k^{\min}(E_e,\omega)}^{x_k^{\max}(E_e,\omega)} dx_k \int_{-1}^{1} dx_\nu \int_0^{2\pi} d\phi_- \\
&\quad \times \frac{E_\nu}{|M - E_e - \omega + |\mathbf{l_e}| x_\nu + \mathbf{k} \cdot \mathbf{n}_\nu|} \sum_{\text{spins}} |\mathcal{M}|^2 |_{p'=p-l_e-l_\nu-k} ,
\end{aligned} \tag{20}$$

where $\phi_- \equiv \phi_k - \phi_\nu$. The lowest photon energy, $\omega^{\rm min}$, is determined by the energy resolution of the detector; thus the total decay rate depends on $\omega^{\rm min}$. We have

$$\omega^{\rm max} = \frac{(M-m_e)^2 - M'^2}{2(M-m_e)} \quad ; \quad E_e^{\rm max}(\omega) = \frac{M^2 + m_e^2 - M'^2 - 2M\omega}{2(M - \omega(1+\beta_e))} \,. \tag{21}$$

The β_e dependence in $E_e^{\rm max}$, noting $\beta_e \equiv |\mathbf{p_e}|/E_e$, implies that $E_e^{\rm max}$ is determined numerically, by iterating to a self-consistent solution for fixed ω. The range in x_k is determined by demanding that $E_\nu \geq 0$, i.e., by requiring

$$(M^2 + m_e^2 - M'^2)\frac{1}{2} + E_e\omega - M(E_e+\omega) - \mathbf{l_e}\cdot\mathbf{k} \geq 0 \,, \tag{22}$$

as well as by demanding that $M - M' - E_e - E_\nu - \omega \geq 0$.

We also compute the polarization of the emitted photon. Defining the polarization states $\epsilon_1^\mu = (0, -\sin\phi_k, \cos\phi_k, 0)$ and $\epsilon_2^\mu = (0, \cos\theta_k\cos\phi_k, \cos\theta_k\sin\phi_k, -\sin\theta_k)$, we can, in turn, define states of circular polarization, namely, $\epsilon_L \equiv (\epsilon_1 + i\epsilon_2)/\sqrt{2}$ and $\epsilon_R \equiv (\epsilon_1 - i\epsilon_2)/\sqrt{2}$. With these conventions, ϵ_L, e.g., does indeed correspond to a left-handed photon when $\mathbf{k} \parallel \mathbf{l_e}$. We define the polarization P via

$$P = \frac{\Gamma_R - \Gamma_L}{\Gamma_R + \Gamma_L} \,. \tag{23}$$

We can also study the polarization as a function of ω and E_e as well; in such cases, we define $P(\omega)$ by replacing $\Gamma_{L,R}$ with $d\Gamma_{L,R}/d\omega$ and $P(\omega, E_e)$ by replacing $\Gamma_{L,R}$ with $d^2\Gamma_{L,R}/d\omega dE_e$.

4. We can now present our results. For definiteness, we specify the input parameters. We use [21, 11]: $G_F = 1.16639 \cdot 10^{-5}\,\mathrm{GeV}^{-2}$, $\alpha^{-1} = 137.03599976$, noting $\alpha = e^2/(4\pi\hbar c)$ in the Heaviside-Lorentz convention, $m_e = 0.510998902\,\mathrm{MeV}$, $m_\pi = 139.57018\,\mathrm{MeV}$, $M = 939.56533\,\mathrm{MeV}$, $M' = 938.27200\,\mathrm{MeV}$, $V_{ud} = 0.9740$, $\Delta_R^V = 0.0240$ [25], $\lambda = 1.267$, $\kappa_v = 3.706$, $\kappa_s = -0.120$, $F_\pi = 92.3\,\mathrm{MeV}$, $g_{\pi NN} = 13.10$, $r_A = 3.395 \cdot 10^{-3}\,\mathrm{MeV}^{-1}$, $M_\Delta = 1232\,\mathrm{MeV}$, $g_{\pi N\Delta} = 1.05$, $b_1 = 12.0$, and the neutron lifetime $\tau_n = 885.7\,\mathrm{s}$. We show the photon energy spectrum $d\Gamma/d\omega$ in Fig. 2, and for the total branching ratio, which depends on the range chosen for ω, we find,

$$\begin{aligned} \omega &\in [0.005\,\mathrm{MeV}, 0.035\,\mathrm{MeV}]\,, & \mathrm{Br}&: 5.17\cdot 10^{-3}\,, \\ \omega &\in [0.035\,\mathrm{MeV}, 0.100\,\mathrm{MeV}]\,, & \mathrm{Br}&: 2.21\cdot 10^{-3}\,, \\ \omega &\in [0.100\,\mathrm{MeV}, \omega^{\rm max} = 0.782\,\mathrm{MeV}]\,, & \mathrm{Br}&: 1.44\cdot 10^{-3}\,. \end{aligned} \tag{24}$$

The branching ratio determined for $\omega \in [0.035\,\mathrm{MeV}, 0.100\,\mathrm{MeV}]$ can be compared directly with the experimental limit of $\mathrm{Br} < 6.9\cdot 10^{-3}$ (90%CL) [6], with which it is compatible. However, the branching ratio for this range of ω, as well as the photon energy spectrum for ω/m_e greater than $\simeq 0.2$, are roughly a factor of two larger than the numerical results reported in Ref. [12]. The discrepancy appears to grow smaller as the photon energy goes to zero. Note, too, that we retain the complete expression for $\sum_{\rm spins}|\mathcal{M}|^2$ in our subsequent numerical calculation; Ref. [12] approximates the integration over phase space and retains the term proportional to $1+3\lambda^2$ only. Note that the approximate angular integrals in Eq. (19) in the first paper of Ref. [12] are correct only if E' (in our notation) is replaced by M', as they neglect $|\mathbf{p}'|$ relative to E'. However, the authors then proceed to integrate over E' in Eq. (20), which is incompatible with the approximation of Eq. (19). We emphasize that the discrepancy is not due to the recoil-order corrections — in Fig. 2 we superimpose the numerical results we find using the leading order form of $\sum_{\rm spins}|\mathcal{M}|^2$, given in Eqs. (13-16). The two curves can scarcely be distinguished; indeed, the recoil-order corrections are no larger than $\mathcal{O}(0.5\%)$. The SSE contribution is itself of $\mathcal{O}(0.1\%)$. In constrast, the recoil-order corrections to the A and a correlations in neutron

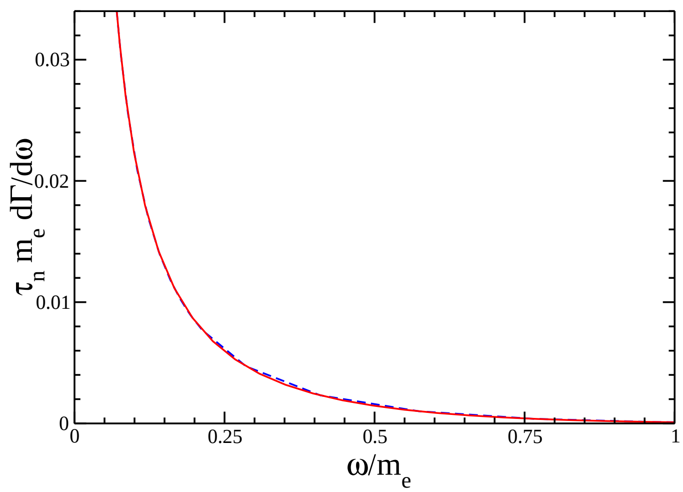

Figure 2: The photon energy spectrum for radiative neutron β–decay. The dashed line denotes the result to NLO in the SSE, whereas the solid line denotes the leading order result, determined using Eqs. (13-16), employed in Ref. [12].

β–decay are of $\mathcal{O}(1-2\%)$ [2]; apparently, the appearance of an additional particle in the final state makes the recoil-order corrections, which are controlled by the dimensionless parameter ϵ, smaller still.

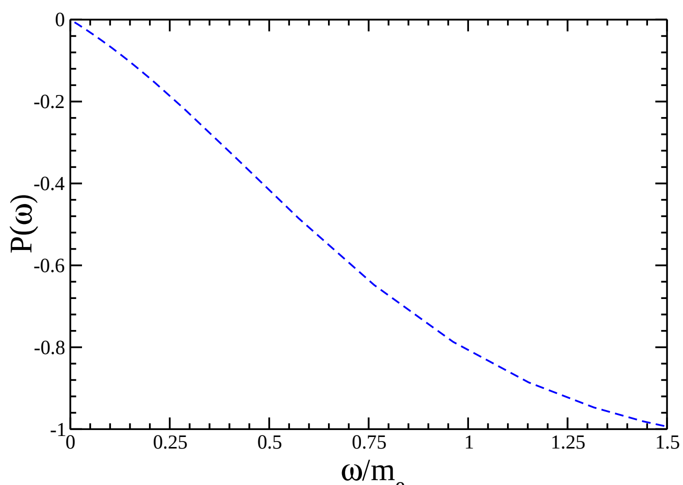

Figure 3: The photon polarization $P(\omega)$ in radiative neutron β–decay to NLO in the SSE.

We present the photon polarization in Fig. 3. The polarization evolves from near-zero at low photon energies to nearly 100% left-handed polarization at high photon energies, as consistent with the discussion of Ref. [15]. The evolution of the polarization with ω is dissected in Fig. 4; as ω grows large, the associated electron momentum is pushed towards zero, and the absolute polarization grows larger. This follows as in the circular basis we can replace $(2\epsilon^*_\pm \cdot l_e - \not{k}\not{\epsilon}^*_\pm)$ in $\langle \bar{\nu}_e e^- \gamma | J^-_\mu | 0 \rangle$ of Eq. (4) with $(2\epsilon^*_\pm \cdot l_e - \omega(1 \pm \gamma_5)\gamma^0 \not{\epsilon}^*_\pm)$ with $\epsilon_{+,-} = \epsilon_{R,L}$. The photon associated with the first term has no circular polarization; this contribution vanishes if $|\mathbf{l_e}| = 0$. In this observable as well the

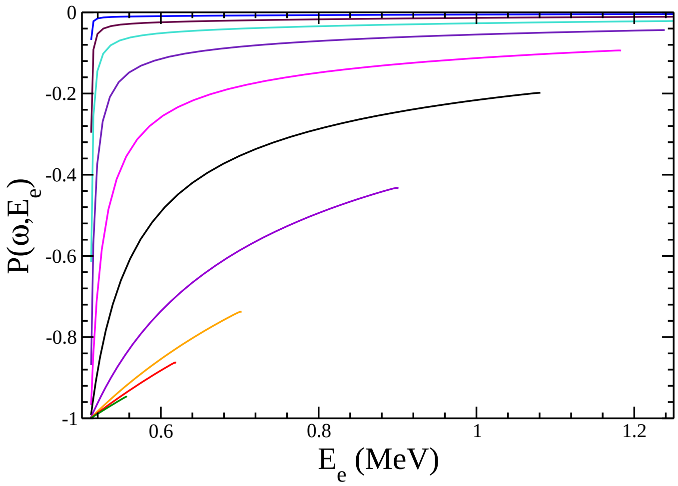

Figure 4: The photon polarization $P(\omega, E_e)$ in radiative neutron β–decay to NLO in the SSE, as a function of E_e for $(E_e^{\max} - E_e)/E_e^{\max} \gtrsim 0.2\%$ and various, fixed ω. For E_e such that $(E_e^{\max} - E_e)/E_e^{\max} \lesssim 0.2\%$, the polarization plunges to -1, see text. The curves from smallest absolute polarization to largest have $\omega = 0.00539, 0.0135, 0.0265, 0.0534, 0.109, 0.209, 0.390, 0.589, 0.673,$ and 0.736 MeV, respectively.

$\mathcal{O}(1/M)$ contributions are $\mathcal{O}(0.5\%)$ or less. Interestingly, the inclusion of these contributions does not impact the determined polarization to an appreciable degree when $\mathbf{l_e} \parallel \pm\mathbf{k}$; $P \approx -1$. Note that as E approaches $E_e^{\max}(\omega)$, $\mathbf{l_e}$ becomes parallel to $-\mathbf{k}$, so that $\epsilon^* \cdot l_e$ approaches zero and P approaches -1 to a high degree of accuracy. In neutron radiative β–decay, the polarization can differ appreciably from unity, so that the calculation of the polarization is *necessary* to realize a SM test; significant deviations from this prediction would nevertheless signify the palpable presence of a left-handed anti-neutrino or of non-$V-A$ currents. As noted by Martin and Glauber [15], the polarization of the photon in S-state orbital electron capture is also sensitive to the *phase* of the vector and axial-vector couplings in the low-energy interaction Hamiltonian [26] if the anti-neutrino is no longer assumed to be strictly right-handed. Such expectations apply to neutron radiative β–decay as well, so that the photon polarization can probe new physics effects to which the correlation coefficients in neutron β–decay are insensitive [27].

5. In this letter, we have computed the photon energy spectrum and photon polarization in neutron radiative β–decay in an effective field theory approach, utilizing heavy baryon chiral perturbation theory and the small-scale expansion, including all terms in $\mathcal{O}(1/M)$. The leading contribution to the photon energy spectrum has been calculated previously [12]; we agree with the expression in Ref. [12] for $\sum_{\rm spins} |\mathcal{M}|^2$, though we disagree with their numerical results for the photon energy spectrum. Moreover, we find that the $O(1/M)$ terms are numerically quite small, generating contributions no larger than $\mathcal{O}(0.5\%)$, so that radiative neutron β–decay is quite insensitive to nucleon structure effects beyond those encoded in g_V and g_A — and offers no clear resolution of the muon radiative capture problem. On the other hand, we have found that nucleon structure effects have a similarly negligible role in the determination of the photon polarization, so that a precise measurement of the photon polarization may well offer a crisp diagnostic of non-SM effects. Such studies may complement other new physics searches. For example, the (pseudo-T-odd) transverse muon polarization $P_\mu^\perp$ in $K^+ \to \mu^+\nu\gamma$ decay is sensitive to large squark generational mixings in generic supersymmetric models [28]

— such charged-current processes survive flavor-changing-neutral-current (FCNC) bounds [28]. The mechanisms discussed in Ref. [28] modify the photon polarization as well, and can also act to modify the $d \to u$ charged, weak current at low energies, to impact the photon polarization, as is our concern here. Finally, we note that the polarization of the photon in radiative B-meson decay, namely in $b \to s\gamma$ decay, is also left-handed in the SM, modulo $\mathcal{O}(1/M_B)$ corrections, estimated to be of order of a few percent; it is also sensitive to non-SM operators [29], as we have discussed here.

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